Technological adoption: Catching-up to the frontier

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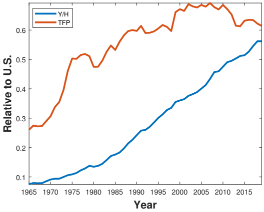
Economic growth: Theory and Empirical Methods

Introduction

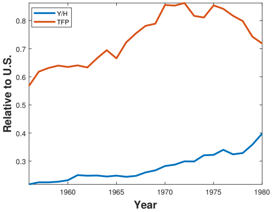
- We have seen a model that describes the discovery of entirely new ideas by researchers at the frontier.
- We have seen in the case of Korea, that much more rapid technological growth is possible for a country far away from the frontier.
- However, even in the case of Korea, we have seen that it did not fully converge.
- It is reasonable to think that implementing an already existing idea is simpler than discovering it anew.
- However, given large TFP differences in the cross-section, convergence taking decades, and convergence often not being complete, implementation cannot be frictionless.

TFP growth

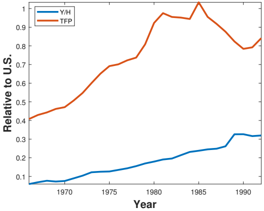
(a) Korea



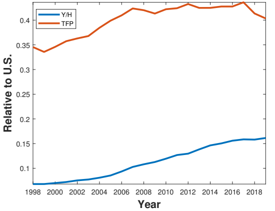
(b) Brazil



(c) Botswana



(d) China



A model of technological adoption

We use the insights from the Romer model and assume output uses differentiated capital goods:

$$Y(t) = L(t)^{1-\alpha} \int_0^{h(t)} x_j(t)^\alpha dj. \quad (1)$$

Here, $h(t)$ is the measure of capital goods that the country knows how to use in period t. This may be different from the technological frontier $A(t)$. For example, the U.S. may know how to produce mRNA vaccines but China does not.

Rewriting the production function

As in the Romer model, each unit of capital goods needs to be produced using a unit of the aggregate available aggregate capital stock:

$$\int_0^{h(t)} x_j(t)dj = K(t). \tag{2}$$

Moreover, each unit is used in the same proportion:

$$x_j(t) = x(t) = \frac{K(t)}{h(t)}. \tag{3}$$

Substituting into the production function:

$$Y(t) = L(t)^{1-\alpha}h(t)x(t)^{\alpha} \tag{4}$$
$$Y(t) = (L(t)h(t))^{1-\alpha}K(t)^{\alpha}. \tag{5}$$

Household income

There is no research sector, instead all labor works in the production sector. The wage and the rental price of capital are

$$w(t) = \frac{\partial Y(t)}{\partial L(t)} = 1 - \alpha \frac{Y(t)}{L(t)} \tag{6}$$

$$r(t) = \frac{\partial Y(t)}{\partial K(t)} = \alpha \frac{Y(t)}{K(t)}. \tag{7}$$

Hence, we have again for household income that

$$\tilde{Y}(t) = w(t)L(t) + r(t)K(t) = Y(t). \tag{8}$$

Accumulation of factors of production

The household sector, as before, accumulates the aggregate capital stock :

$$\dot{K}(t) = sY(t) - \delta K(t), \tag{9}$$

The population grows at rate n:

$$\dot{L}(t) = nL(t). \tag{10}$$

Skill accumulation

Key to the model is the accumulation of the skill level h. Skills evolve over time according to

$$\dot{h}(t) = \mu \exp(\psi u) A(t)^{\gamma} h(t)^{1-\gamma} \tag{11}$$

- As in the model of human capital accumulation, u measures the time people spend learning new skills. This may be explicit research activity or learning-by-doing.
- The current level of skills makes it easier to learn new skills, i.e., $\gamma < 1$.
- A further developed technological frontier makes it easier to lean new skills, i.e, $\gamma > 0$.

Rewrite in growth rates

$$\frac{\dot{h}(t)}{h(t)} = \mu \exp(\psi u) \left(\frac{A(t)}{h(t)} \right)^{\gamma} \tag{12}$$

The speed of adoption depends on the distance to the technological frontier. When that distance is large, $\frac{A}{h}$ is large, skill accumulation growth is fast. I.e., it is easier to copy technologies that are already in use for a long time. Finally, we assume that the frontier grows according to

$$\frac{\dot{A}(t)}{A(t)} = g. \tag{13}$$

Steady state

Given the production function, we know already that the capital-to-output ratio solves for:

$$z^* = \left(\frac{K(t)}{Y(t)}\right)^* = \frac{s}{n + g_h(t) + \delta}, \tag{14}$$

which has a steady state if $g_h(t) = g_h$, a constant, in steady state. Hence, in steady state, output per worker is

$$\left(\frac{Y(t)}{L(t)}\right)^* = \left(\frac{s}{n + g_h(t) + \delta}\right)^{\frac{\alpha}{1-\alpha}} h(t). \tag{15}$$

Next, we derive g_h and $h(t)$.

Skill growth in steady state

For the growth rate of skills

$$\frac{\dot{h}(t)}{h(t)} = \mu \exp(\psi u) \left(\frac{A(t)}{h(t)} \right)^{\gamma} \tag{16}$$

to be constant, we directly see that $\frac{A(t)}{h(t)}$ needs to be constant, i.e., $g_h = g$.

The distance to the frontier in steady state

$$\frac{\dot{h}(t)}{h(t)} = \mu \exp(\psi u) \left(\frac{A(t)}{h(t)} \right)^{\gamma} \tag{17}$$

$$g = \mu \exp(\psi u) \left(\frac{A(t)}{h(t)} \right)^{\gamma} \tag{18}$$

$$\frac{h(t)}{A(t)} = \left(\frac{\mu \exp(\psi u)}{g} \right)^{\frac{1}{\gamma}} . \tag{19}$$

Improved education pushes the economy closer to the technological frontier.

Obtaining $h(t)$

To obtain the level of skills, we need to solve the differential equation of skill accumulation:

$$\dot{h}(t) = \mu \exp(\psi u) A(t)^{\gamma} h(t)^{1-\gamma}, \tag{20}$$

To solve the equation, define

$$v(t) = h(t)^{\gamma} \tag{21}$$

with

$$\dot{v}(t) = \dot{h}(t) \gamma h(t)^{\gamma-1} \tag{22}$$

Substituting yields

$$\dot{v}(t) = \gamma \mu \exp(\psi u) A(t)^{\gamma}, \tag{23}$$

Obtaining $h(t)$ II

Substituting for $A(t) = A(0)\exp(gt)$ yields:

$$\dot{v}(t) = \gamma\mu\exp(\psi u)A(0)^{\gamma}\exp(\gamma gt) \tag{24}$$

with solution

$$v(t) = \frac{1}{g}\mu\exp(\psi u)A(0)^{\gamma}\exp(\gamma gt) + I \tag{25}$$

Evaluating at $v(0)$:

$$I = v(0) - \frac{1}{g}\mu\exp(\psi u)A(0)^{\gamma} \tag{26}$$

Plugging in and substituting again $v(t) = h(t)^{\gamma}$ yields:

$$h(t)^{\gamma} = \frac{1}{g}\mu\exp(\psi u)A(0)^{\gamma}\exp(\gamma gt) + h(0)^{\gamma} - \frac{1}{g}\mu\exp(\psi u)A(0)^{\gamma} \tag{27}$$

Obtaining $h(t)$ III

Solving for $h(t)$

$$h(t) = \left(h(0)^{\gamma} + \frac{1}{g}\mu \exp(\psi u) A(0)^{\gamma} \left[\exp(\gamma g t) - 1\right]\right)^{\frac{1}{\gamma}} \tag{28}$$

- A higher initial skill level increases skills today.
- More and better education increases the skill level.
- A higher initial level of the of technology in the rest of the world increases skills.

Output per worker in steady state

$$\left(\frac{Y(t)}{L(t)}\right)^* = \left(\frac{s}{n+g_h+\delta}\right)^{\frac{\alpha}{1-\alpha}} \left(h(0)^\gamma + \frac{1}{g}\mu \exp(\psi u) A(0)^\gamma \left[\exp(\gamma g t) - 1\right]\right)^{\frac{1}{\gamma}} \quad (29)$$

- Higher capital per worker leads to higher output per worker. Hence, for a developing economy, population growth decreases output per worker unambiguously as it no longer affects $A(t)$.
- The model provides a reason why education leads to higher output per worker. Education does not make workers more productive in the abstract, it makes them more productive by allowing them to use more advanced capital goods.
- Pushing out the technological frontier benefits all countries as they can copy those new ideas over time.

Skill accumulation outside the steady state

We can use the solution for the level of skills to solve for the transition dynamics:

$$\frac{\dot{h}(t)}{h(t)} = \mu \exp(\psi u) \left(\frac{A(t)}{h(t)} \right)^{\gamma} \tag{30}$$

$$h(t) = \left(h(0)^{\gamma} + \frac{1}{g} \mu \exp(\psi u) A(0)^{\gamma} \left[\exp(\gamma g t) - 1 \right] \right)^{\frac{1}{\gamma}} \tag{31}$$

Combining the equations and rewriting:

$$\frac{\dot{h}(t)}{h(t)} = \mu \exp(\psi u) \frac{A(t)^{\gamma}}{h(0)^{\gamma} + \frac{1}{g} \mu \exp(\psi u) A(t)^{\gamma} - \frac{1}{g} \mu \exp(\psi u) A(0)^{\gamma}} \tag{32}$$

$$= g \frac{\mu \exp(\psi u) A(t)^{\gamma}}{h(0)^{\gamma} g + \mu \exp(\psi u) A(t)^{\gamma} - \mu \exp(\psi u) A(0)^{\gamma}} \tag{33}$$

Skill accumulation outside the steady state II

$$\frac{\dot{h}(t)}{h(t)} = \frac{g}{1 + \left(\frac{h(0)}{A(t)}\right)^{\gamma} \frac{g}{\mu \exp(\psi u)} - \left(\frac{A(0)}{A(t)}\right)^{\gamma}} \tag{34}$$

- When starting in steady state: $\frac{h(0)}{A(0)} = \left(\frac{\mu \exp(\psi u)}{g}\right)^{1/\gamma}$ and $A(0) = A(t)$, $\frac{\dot{h}(t)}{h(t)} = g$.
- The lower $\frac{h(0)}{A(0)}$ is relative to its steady state, the faster is the initial growth rate.
- As time passes, $A(t)$ grows and $\frac{\dot{h}(t)}{h(t)} \mapsto g$.
- For any $h(0), A(0)$, increasing μ, ψ, u, or g increases $\frac{\dot{h}(t)}{h(t)}$.

Transition dynamics for output per worker

As before, let us write output as a function of the capital to output ratio

$$Y(t) = h(t)L(t)\left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \tag{35}$$

$$y(t) = h(t)\left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} \tag{36}$$

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{h}(t)}{h(t)} + \frac{\alpha}{1-\alpha}\frac{\dot{z}(t)}{z(t)} \tag{37}$$

with

$$\frac{\dot{z}(t)}{z(t)} = (1-\alpha)\frac{s}{z(t)} - (1-\alpha)\left(n + \frac{\dot{h}(t)}{h(t)} + \delta\right). \tag{38}$$

Transition dynamics for output per worker II

$$\frac{\dot{y}(t)}{y(t)} = \frac{\dot{h}(t)}{h(t)} + \alpha \left[\frac{s}{z(t)} - \left(n + \frac{\dot{h}(t)}{h(t)} + \delta \right) \right]. \tag{39}$$

Similar to the Romer model, a temporary increase in $\frac{\dot{h}(t)}{h(t)}$ increases the growth rate of $y(t)$ but it increases it initially by less than the increase in $\frac{\dot{h}(t)}{h(t)}$ because the capital to output ratio declines initially.

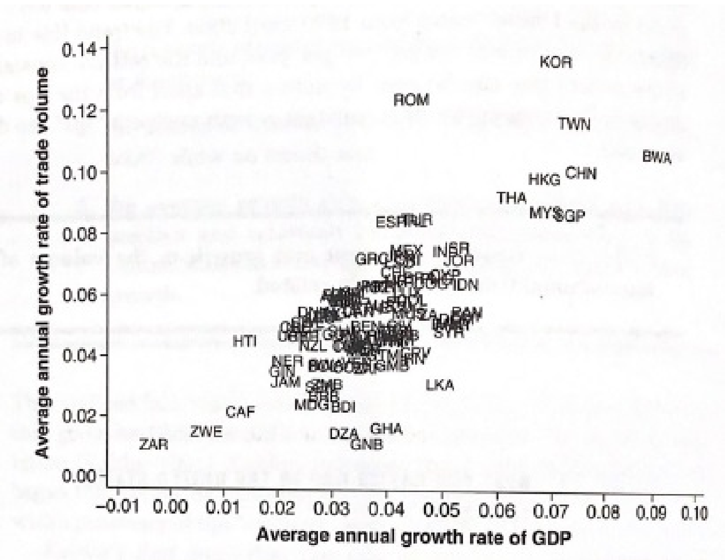
Knowledge spillover and trade

Knowledge spillover and trade

So far, we assume that acquiring the knowledge to use more advanced capital goods is the only way to obtain those. However, there are good reasons to believe that trade is a way for new technologies becoming available to countries:

- A famous example is China severely restricting trade in the 14th century which coincides with its downfall as technological leader.
- The trade embargo on modern Iran coincides with Iran losing in terms of output per worker.
- FDI into China was an important source of technological adoption.
- Patenting may assures that recent inventions can only be bought from aboard.

The data suggests that a link exists



The data suggests that a link exists II

Table 3: Bilateral trade linkages and the European MTC

	GDP per working age population			
Bilateral exports US	France	Germany	Italy	Spain
Lag 0	0.6509*	0.7485*	0.7825*	0.7994*
Lag 1	0.7058*	0.7670*	0.7938*	0.7640*
Lag 2	0.7079*	0.7521*	0.7643*	0.6943*
Lag 3	0.6798*	0.7136*	0.7066*	0.6091*
	Investment per working age population			
Bilateral exports US	France	Germany	Italy	Spain
Lag 0	0.6150*	0.7905*	0.4726*	0.7965*
Lag 1	0.6222*	0.7340*	0.4917*	0.7770*
Lag 2	0.5733*	0.6283*	0.4702*	0.7184*
Lag 3	0.5001*	0.5012*	0.4287*	0.6504*
	Relative price of capital			
Bilateral exports US	France	Germany	Italy	Spain
Lag 0	-0.6575*	-0.1974*	-0.7331*	-0.5444*
Lag 1	-0.6416*	-0.2554*	-0.6604*	-0.4967*
Lag 2	-0.6218*	-0.3134*	-0.5667*	-0.4378*
Lag 3	-0.5838*	-0.3694*	-0.4522*	-0.3617*

Source: López and de Blas Pérez (2018)

Changing the production function

Assume h capital goods are produced at home and m are imported:

$$Y(t) = L(t)^{1-\alpha} \int_0^{h(t)+m(t)} x_j(t)^\alpha dj. \tag{40}$$

Home production and trade

A country produces $z(t)$ units of each home-based capital good and, hence,:

$$z(t)h(t) = K(t). \tag{41}$$

The country keeps only $x(t)h(t)$ of these goods and buys $x(t)$ of the imported goods. Hence, balanced trade implies:

$$x(t)m(t) = K(t) - x(t)h(t) \tag{42}$$
$$K(t) = x(t)[m(t) + h(t)]. \tag{43}$$

Rewriting the production function

Equal input usage implies

$$Y(t) = L(t)^{1-\alpha} \int_0^{h(t)+m(t)} x_j(t)^\alpha dj \tag{44}$$

$$Y(t) = L(t)^{1-\alpha}(h(t) + m(t))x(t)^\alpha. \tag{45}$$

Using the trade balance:

$$Y(t) = L(t)^{1-\alpha}(h(t) + m(t)) \left(\frac{K(t)}{m(t) + h(t)} \right)^\alpha \tag{46}$$

$$Y(t) = ([m(t) + h(t)]L(t))^{1-\alpha} K(t)^\alpha \tag{47}$$

Steady state

Define $N(t) = h(t) + m(t)$ and consider again the growth rate of the capital-to-output ratio, and the capital accumulation equation:

$$z(t) = \frac{K(t)^{1-\alpha}}{(N(t)L(t))^{1-\alpha}} \quad \frac{\dot{z}(t)}{z(t)} = 1 - \alpha\frac{\dot{K}(t)}{K(t)} - 1 - \alpha\left(n + \frac{\dot{N}(t)}{N(t)}\right) \tag{48}$$

$$\frac{\dot{z}(t)}{z(t)} = \frac{s}{z(t)} - \delta. \tag{49}$$

Combining the equations and assuming that a steady state exists:

$$n + \frac{\dot{N}(t)}{N(t)} = \frac{s}{z^*} - \delta \tag{50}$$

Steady state II

$$z^* = \frac{s}{n + \frac{\dot{N}(t)}{N(t)} + \delta}, \tag{51}$$

which is constant if $\frac{\dot{N}(t)}{N(t)} = g_N(t)$ is constant. One can show, that, in steady state, $g_N(t) = g$, i.e., a constant.

Output per worker in steady state

From the production function:

$$Y(t)^{1-\alpha} = \left(\frac{K(t)}{Y(t)}\right)^{\alpha} ([h(t) + m(t)]L(t))^{1-\alpha} \quad (52)$$

$$Y(t) = \left(\frac{K(t)}{Y(t)}\right)^{\frac{\alpha}{1-\alpha}} [h(t) + m(t)]L(t) \quad (53)$$

$$(54)$$

In steady state:

$$Y(t)^{*} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} [h(t) + m(t)]L(t) \quad (55)$$

$$y(t)^{*} = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} [h(t) + m(t)] \quad (56)$$

The steady state II

$$y(t)^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} [h(t) + m(t)] \quad (57)$$

$$(58)$$

Income per worker is increasing in the number of imported varieties of capital goods. Trade makes us richer as it allows us access to capital goods currently not produced in our country.

Trade and human capital accumulation

$$y(t)^* = \left(\frac{s}{n+g+\delta}\right)^{\frac{\alpha}{1-\alpha}} [h(t) + m(t)] \tag{59}$$

- Trade can be a substitute for human capital investment. By selling "unsophisticated" capital goods, the country can gain access to more advanced capital goods. Note, the implications would be somewhat different in a Schumpeterian model.
- China is a good example. By foreign countries bringing their technologies with them and using an uneducated Chinese workforce, output was able to grow at tremendous rates.

References

López, M. C. and B. de Blas Pérez (2018): "Faraway, so close!, technology diffusion and firm heterogeneity in the medium term cycle of advanced economies," *Documentos de trabajo del Banco de España*, 1–64.